SOLUTIONS

Solution 1 :

P : $y^2 = 2px$

$M \in D \Leftrightarrow \det(\overrightarrow{FM}, \alpha\vec{i} + \vec{j}) = 0 \Leftrightarrow \begin{vmatrix} x - \frac{p}{2} & \alpha \\ y & 1 \end{vmatrix} = 0 \Leftrightarrow x - \frac{p}{2} - \alpha y = 0$;

D: $x - \alpha y - \frac{p}{2} = 0$

1°/ $M \in P \cap D \Leftrightarrow \begin{cases} M(x,y) \ ; \ y^2 = 2px \\ x - \alpha y - \frac{p}{2} = 0 \end{cases}$

on a : $y^2 - 2p(\alpha y + \frac{p}{2}) = 0 \Leftrightarrow y^2 - 2p\alpha y - p^2 = 0$

$\Delta = 4p^2\alpha^2 + 4p^2 = 4p^2(\alpha^2 + 1)$

$y' = \frac{2p\alpha - 2p\sqrt{\alpha^2+1}}{2} = p(\alpha - \sqrt{\alpha^2+1})$; $y'' = p(\alpha + \sqrt{\alpha^2+1})$.

$\Rightarrow x' = \alpha y' + \frac{p}{2} = \alpha p(\alpha - \sqrt{\alpha^2+1}) + \frac{p}{2}$ d'où $x' = p(\alpha^2 - \alpha\sqrt{\alpha^2+1} + \frac{1}{2})$

$x'' = \alpha y'' + \frac{p}{2} = \alpha p(\alpha + \sqrt{\alpha^2+1}) + \frac{p}{2} = p(\alpha^2 + \alpha\sqrt{\alpha^2+1} + \frac{1}{2})$

Soit $K = M' * M''$; $x_K = \frac{x' + x''}{2} = p(\alpha^2 + \frac{1}{2})$; $y_K = \frac{y' + y''}{2} = p\alpha$

d'où $K(p\alpha^2 + \frac{p}{2}, p\alpha)$

2°/ Soit $x = p\alpha^2 + \frac{p}{2}$ et $y = p\alpha$

$\begin{cases} x = p(\alpha^2 + \frac{1}{2}) \\ y = p\alpha \end{cases}$

Lorsque α décrit $\mathbf{R} \Leftrightarrow y$ décrit $\mathbf{R}$ d'où

$x = p(\frac{y^2}{p^2} + \frac{1}{2}) \Leftrightarrow x = \frac{y^2}{p} + \frac{1}{2}p \Leftrightarrow y^2 = p(x - \frac{1}{2}p)$